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Magnetic field generation from Alfvén waves

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Abstract : A static induced magnetization of Alfvén waves (IMAW) propagating along helical lines of force on an ambient magnetic field has been defined and evaluated. This magnetization is due to the inverse Faraday effect (IFE) and follows from the magnetic moment of ordered gyrating motion of charges in presence of electromagnetic (EM) waves. The IMAW is expected to be important in the physics of sun and pulsars and also of laboratory devices for generation of plasmas and their heating.

Keywords : Alfvén waves, inverse Faraday effect.

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1. Introduction

Alfvén waves [1] are the low frequency limit of EM waves propagating parallel to a static magnetic field in a two component plasma. Here we are considering a plasma of zero resistivity, valid at frequencies well below the ion cyclotron frequency [2]. A static magnetic field is found to be generated from Alfvén waves propagating along helical magnetic lines of force in cold magnetized plasmas. This field is a consequence of wave particle interaction and is due to the inverse Faraday effect (IFE) [3,4].

The IFE is the generation of static magnetization from the cumulative effect of magnetic moment of ordered loop motion of charges due to waves of finite amplitude in different material media, including plasmas. Its lines follow the locus of the centres of curvature of these loop motions, which is also the path of rays of the waves. In the simplest case it is due to ordered circular motion of charges in presence of a circularly polarized EM wave of rectilinear propagation. For the purpose of evaluation of this IFE, the plasma is regarded as a dense collection of charges of both sign maintaining a high order of charge neutrality.

For strong high frequency EM waves, the induced magnetization of IFE is formed mainly from the electron current because then the ion current is not a dominating factor for fields of moderate or weak intensity, and is not significantly affected by the ambient field. But, for Alfvén waves, which occur in the lowest frequency limit of EM waves in magnetized plasmas, the situation is different in this limit, the ions no longer remain the nonparticipating background constituent for charge neutrality ; rather, the role of ionic motion becomes dominant. Hence, we have considered both electron and ion motion.

Alfvén waves are supposed to be the main heating agencies of the solar corona. The surface Alfvén waves deposit energy in a thin resonance region and several such regions together provide the heat required to raise the solar corona temperature to million degrees.

2. Geometry of the problem

Often in astrophysical problems the ambient magnetic field $\boldsymbol{H}$ is everywhere parallel to the current $\boldsymbol{J}$ $\left(= \frac{c}{4\pi} \nabla \times \boldsymbol{H}\right)$ of charges. These are force-free fields since associated Lorentz force $(= \boldsymbol{J} \times \boldsymbol{H}/c)$ is identically zero. For any force-free field there exists a scalar function of position $\alpha(\boldsymbol{r})$ such that

$$\nabla \times \boldsymbol{H} = \alpha \boldsymbol{H} \tag{1}$$

in addition to the solenoidal condition

$$\nabla \cdot \boldsymbol{H} = 0. \tag{2}$$

Relations (1) and (2) give

$$(\boldsymbol{H} \cdot \nabla)\alpha = 0, \tag{3}$$

α is therefore constant along the magnetic field lines. If these lines cover surfaces then α must be constant in each such surface. A particularly simple situation is that in which α is constant everywhere ; in this case curl of eq. (3) immediately gives

$$\left(\nabla^2 + \alpha^2\right) \boldsymbol{H} = 0. \tag{4}$$

The simplest example of a force-free field, with constant α, in cartesian coordinates, is

$$\boldsymbol{H} = H_0 \,(\sin\,\alpha z,\ \cos\,\alpha z,\ 0) \tag{5}$$

where H_0 is a constant. The $\boldsymbol{H}$ lines, as indicated in Figure 1 [5] lie in the X–Y plane and their direction rotates with increasing z in a sense that is left handed or right handed according as α is positive or negative. The vector potential of $\boldsymbol{H}$ is simply $\boldsymbol{A} = \alpha^{-1} \boldsymbol{H}$, so that the helicity density is uniform :

$$\boldsymbol{A} \cdot \boldsymbol{H} = \alpha^{-1} H^2 = \alpha^{-1} H_0^2 \tag{6}$$

We know that resistivity $(1/\sigma)$ dissipates magnetic helicity at a rate proportional to $\frac{\boldsymbol{J}.\boldsymbol{H}}{\sigma}$ [4] where $\boldsymbol{J}$ is the surface density of current and σ is the conductivity of the fluid. Here, we have considered plasma of zero resistivity ; and there is no dissipation of magnetic helicity. Hence, the helicity is conserved which also follows from eq. (6).

We have employed the system of cartesian coordinates. Of course, we can do some complicated problems using cylindrical coordinates which are possibly more useful. One such problem is also under our consideration and would soon be published elsewhere.

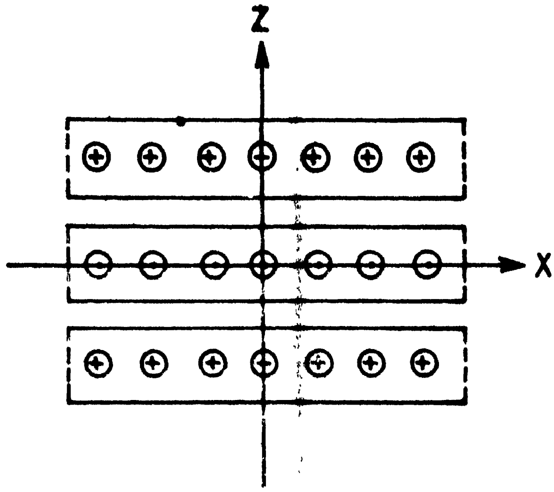

Figure 1. Lines of force of the field (5) (with $\alpha > 0$) indicates a line in the positive y direction, into the paper, ⊕ indicates a line in the negative y–direction , the lines of force rotate in a left handed sense with increasing Z, [5].

3. Formulation of the problem and the preliminary analysis

Any charged particle of the species δ, having charge q per particle and number density N, gyrating with velocity u_δ, contributes a magnetic dipole moment

$$\mu_d = \frac{1}{2c}(r_d \times j_d) \tag{7}$$

where r_δ is the position vector of the charge q_δ with respect to a fixed origin, j_d ($= N_d\, q_d\, u_d$) is the corresponding surface density of current. The induced magnetic field, denoted by H_i, is given by

$$H_i = \sum_\delta 4\, \pi N_\delta\, \mu_\delta$$

In a neutral, two component, plasma, since $N_e = N_i = N_0$, we find that

$$H_i \;=\; 4\, \pi N_0\, (\mu_e + \mu_i) \tag{8}$$

where μ_e is the magnetic dipole moment due to motion of electrons and μ_i is that due to ionic motion.

The equations of motion of an electron and an ion in an EM field are

$$\dot{u}_e = -\frac{e}{m}E - \frac{e}{mc}(u_e \times H) \tag{9}$$

$$\dot{u}_i = \frac{e}{m}E + \frac{e}{mc}(u_i \times H) \tag{10}$$

where u_e is the velocity and m is the mass of an electron E is the electric intensity vector, M is the mass of an ion and u_i is the ionic velocity.

The constraints are

(i) Applied electric field $E = (E_x, E_y, 0)$.

(ii) The ambient magnetic field H with components $H_x = H_0 \sin \alpha z$, $H_y = H_0 \cos \alpha z$, $H_z = 0$. The applied electric field may be written as

$$E_x = ae^{i\phi}, \quad E_y = iae^{i\phi}, \quad E_z = 0 \tag{11}$$

where $\phi = kz - \omega t$, k is the wave propagation vector and ω is the wave frequency, a is the amplitude of the applied field.

Solution of eq. (9) may be written as

$$\operatorname{Re} x_e = \frac{ea}{A_e m}\left[\left(\omega^2 - \Omega_{x_e}{}^2\right)\cos\phi + \Omega_{x_e}\Omega_{y_e}\sin\phi\right] \tag{12}$$

$$\operatorname{Re} y_e = \frac{ea}{A_e m}\left[-\left(\omega^2 - \Omega_{y_e}{}^2\right)\sin\phi - \Omega_{x_e}\Omega_{y_e}\cos\phi\right] \tag{13}$$

$$\operatorname{Re} z_e = \frac{ea\omega}{A_e m}\left[\Omega_{y_e}\sin\phi - \Omega_{x_e}\cos\phi\right] \tag{14}$$

where Re stands for the real part, and

$$A_e = \left(\omega^2 - \Omega_{x_e}\right)\left(\omega^2 - \Omega_{y_e}{}^2\right) - \Omega_{x_e}{}^2\Omega_{y_e}{}^2$$

$$\Omega_{x_e} = \frac{eH_x}{mc}, \qquad \Omega_{y_e} = \frac{eH_y}{mc}.$$

Solution of eq. (10) may be written as

$$\operatorname{Re} x_i = \frac{ea}{A_i M}\left[\Omega_{x_i}\Omega_{y_i}\sin\phi + \left(\omega^2 - \Omega_{x_i}{}^2\right)\cos\phi\right] \tag{15}$$

$$\operatorname{Re} y_i = \frac{ea}{A_i M}\left[\Omega_{x_i}\Omega_{y_i}\cos\phi + \left(\omega^2 - \Omega_{y_i}{}^2\right)\sin\phi\right] \tag{16}$$

$$\operatorname{Re} z_i = \frac{ea\omega}{A_i M}\left[\Omega_{x_i}\cos\phi - \Omega_{y_i}\sin\phi\right] \tag{17}$$

where $\Omega_{x_i} = \frac{eH_x}{Mc}$, $\Omega_{y_i} = \frac{eH_y}{Mc}$ and $A_i = \left(\omega^2 - \Omega_{x_i}{}^2\right)$.

$$\left(\omega^2 - \Omega_{y_i}{}^2\right) - \Omega_{x_i}{}^2\Omega_{y_i}{}^2.$$

The components of the static part of the induced magnetization, evaluated from the field solutions (12) – (17), are

$$\langle H_{ix}\rangle = -\frac{4\pi Ne}{2c}e^2a^2\omega^4\left[\frac{\Omega_{x_i}}{A_i^2M^2} + \frac{\Omega_{x_i}}{A_e^2m^2}\right] \tag{18}$$

$$\langle H_{iy}\rangle = -\frac{4\pi Ne}{2c}e^2a^2\omega^4\left[\frac{\Omega_{y_i}}{A_i^2M^2}+\frac{\Omega_{y_e}}{A_e^2m^2}\right] \tag{19}$$

$$\langle H_{iz}\rangle = \frac{4\pi Ne}{2c}e^2a^2\omega^3\left[\frac{\omega^2-\Omega_{x_i}{}^2-\Omega_{y_i}{}^2}{A_i^2M^2}-\frac{\omega^2-\Omega_{x_e}{}^2-\Omega_{y_e}{}^2}{A_e^2m^2}\right]. \tag{20}$$

The symbol < > stands for the static part such that $\langle P\rangle \left(=\frac{l}{T}\int_0^T pdT\right)$ is the average of the quantity P over the wave time period $T\left(=\frac{2\pi}{\omega}\right)$.

4. The Alfven wave approximation

The *Alfvén* wave approximation is $|\Omega_i| >> |\omega|$. Using this approximation the induced magnetic field components are found to be

$$\langle H_{ix}\rangle = -\frac{4\pi Ne}{2c} - \frac{e^2a^2\cdot My_{ex}}{\omega^3m^3\left(y_{ex}^2+y_{ey}^2\right)^2} \tag{21}$$

$$\langle H_{iy}\rangle = -\frac{4\pi Ne}{2c}\frac{e^2a^2\cdot My_{ey}}{\omega^3m^3\left(y_{ex}^2+y_{ey}^2\right)^2} \tag{22}$$

$$\langle H_{iz}\rangle = 0 \tag{23}$$

where $\quad y_{ex}=\frac{\Omega_{ex}}{\omega},\qquad y_{ey}=\frac{\Omega_{ey}}{\omega}.$

Numerical estimation

For the solar atmosphere [1,6]

$$\omega_{P_e}=10^{11}\,/\text{sec},\quad \omega=10^{9}\,/\text{sec},\quad H_0=10^{11}\,\text{Gauss},\quad a=10^{2}\,\text{e.s.u.}$$

the induced magnetic field is

$$\langle H_i\rangle \cong 10^5 \text{ Gauss.}$$

5. Profile of the static pressure

Here, we consider the case where the pressure gradient is exactly balanced by the Lorentz force.

So,

$$-\nabla p+\frac{1}{4\pi}(\nabla\times \boldsymbol{H})\times \boldsymbol{H}=0. \tag{24}$$

For the force-free ambient field since $(\nabla\times \boldsymbol{H})\times \boldsymbol{H}=0$, we find that $\nabla p = 0$; so the corresponding pressure is constant. On the other hand, if the induced magnetic field H_i is also included, then eq. (24) is replaced by

$$-\nabla p+\frac{1}{4\pi}[\nabla\times(\boldsymbol{H}+\boldsymbol{H}_i)]\times(\boldsymbol{H}+\boldsymbol{H}_i)=0 \tag{25}$$

because the effective static magnetic field is now $\boldsymbol{H}+\boldsymbol{H}_i$ and here $\nabla p\neq 0$.

6. Some other parameters

(a) Current J_{ob} due to observed field $(H+H_i)$ may be expressed as

$$J_{ob}=\frac{c\alpha}{4\pi}\sqrt{\left[(H_x+H_{ix})^2+(H_y+H_{iy})^2\right]}\,. \tag{26}$$

Putting values of H_x, H_y from relation (5) and H_{ix}, H_{iy} from relations (21) and (22), respectively, and knowing α we can estimate $\langle J_{ob}\rangle$.

(b) Angle between the ambient field H and the observed field $(H+H_i)$, is given by

Since, $\qquad \beta=\cos^{-1}\dfrac{(H+H_i)\cdot H}{|H+H_i|\cdot|H|}$,

we have

$$\beta=\cos^{-1}\frac{H_x^2+H_xH_{ix}+H_y^2+H_yH_{iy}}{\left[(H_x+H_{ix})^2+(H_y+H_{iy})^2\right]^{1/2}\left[H_x^2+H_y^2\right]^{1/2}}. \tag{27}$$

Using the relation (5) for H_x, H_y and relations (21) and (22) respectively for H_{ix} and H_{iy} we can estimate β.

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